ME 423: FLUIDS ENGINEERING

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Lecture-19 (11/11/2024)

Hydraulics of Pipeline Systems

(Transient System / Unsteady flow)

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Compressible flow in an elastic Pipe

Problem 11.48 (Potter)

Gasoline (SG = 0.68) is supplied by gravity without pumping from a storage tank through a 800-m-long 50-mm diameter nearly horizontal pipe into a tanker truck. There is a quick-acting valve at the end of the pipe. The difference in elevations of gasoline between the reservoir and the truck tank is 8 m. The valve is operated at a position where K = 5. At a particular situation, the operator decided to suddenly close the valve. Assuming that the gasoline is slightly compressible, and the piping is elastic, determine:

(a) The acoustic wave speed.

(b) The pressure rise at the valve once the valve is rapidly closed in a manner to cause water hammer.

(c) If the maximum allowable pressure in the pipe is 250 kPa, what can you conclude about the outcome of the water (gasoline) hammer activity?

The pipe is made of aluminum, E = 70 GPa, with 2.5-mm-thick walls, and the bulk modulus of elasticity of gasoline is B = 1.05 GPa. Assume that f = 0.015.

Solution:

(a) Acoustic wave speed in Gasoline is given be:

$$a = \sqrt{\frac{B/\rho}{1 + (D/e)(B/E)}} \quad \rightarrow a = \sqrt{\frac{1.05 \times 10^9/(0.68 \times 1000)}{1 + \left(\frac{50}{2.5}\right)\left(\frac{1.05 \times 10^9}{70 \times 10^9}\right)}} \quad \rightarrow a = 1090\ \text{m/s}$$

(b) Initial steady-state velocity is, with K_{ss} = 5 :

$$V_{ss} = \sqrt{\frac{2g(H_1 - H_3)}{fL/D + K}}$$

$$V_{ss} = \sqrt{\frac{2\times 9.81\times 8}{\frac{0.015\times 800}{0.05}+5}} = 0.800 \text{ m/s}$$

Since valve closes rapidly, the final velocity, V = 0

Hence the pressure rise at the valve due to water (gasoline) hammer is:

$$\Delta p = -\rho a \Delta V = -(0.68\times 1000)(1090)(0-0.8) = 593 \text{ kPa}$$

(c) Initial steady-state pressure at the valve is :

$$\text{Energy equation: } H_1 = H_3 + f\frac{L}{D}\frac{V^2}{2g} \qquad \rightarrow Z_1 = \left(Z_3 + \frac{p_3}{\gamma}\right) + f\frac{L}{D}\frac{V^2}{2g}$$

$$\rightarrow p_3 = \gamma\left[(Z_1 - Z_3) - f\frac{L}{D}\frac{V^2}{2g}\right]$$

$$\rightarrow p_3 = (0.68\times 1000\times 9.81)\left[(8) - (0.015)\frac{800}{0.05}\frac{0.8^2}{2(9.81)}\right]$$

$$p_3 = 1142 \text{ Pa}$$

Hence the instantaneous pressure at valve will be: $p = p_3 + \Delta p = 594 \text{ kPa}$

Which is over two times of the allowable pressure in the pipe (250 kPa). It is possible that the pipe could rupture.

Compressible flow in an elastic Pipe

Problem 11.47 (Potter)

Water (B = 2.2 GPa) is flowing through a pipe whose diameter is 200 mm and whose length is 800 m. The pipe is made of cast iron (E = 150 GPa) with a thickness of 12 mm. There is a reservoir at the upstream end of the pipe and a valve at the downstream end. Under steady-state conditions the discharge is $Q = 0.05\ m^3/s$, when a valve at the end of the pipe is actuated very rapidly so that water hammer occurs.

(a) How long does it take for an acoustic wave to travel from the valve to the reservoir and back to the valve?

(b) Determine the change in pressure at the valve if the valve is opened such that the discharge is doubled.

(c) Determine the change in pressure at the valve if the valve is closed such that the discharge is halved.

Solution:

(a) Acoustic wave speed in water bounded in a pipeline is given be:

$$a = \sqrt{\frac{B/\rho}{1 + (D/e)(B/E)}} \quad \rightarrow a = \sqrt{\frac{2.2\times 10^9/(1000)}{1 + \left(\frac{200}{12}\right)\left(\frac{2.2\times 10^9}{150\times 10^9}\right)}} \quad \rightarrow a = 1330\ \text{m/s}$$

Wave travel time from valve to reservoir = L/a = 800/1330 = 0.6 s

(b) Valve is rapidly opened such that the discharge is doubled:

$$\Delta V = \frac{Q}{\pi D^2/4} = \frac{0.05}{0.7854 \times 0.2^2} = 1.592 \text{ m/s}$$

Final velocity is doubled to initial velocity.
Change of velocity is +ve

$$\therefore \Delta p = -\rho a \Delta V = -1000 \times 1330 \times 1.592 = \underline{-2.12 \times 10^6 \text{ Pa, or } -2120 \text{ kPa}}$$

Note the large pressure reduction due to the water hammer effect. The original pressure at the valve must be sufficiently large so that cavitation will not occur. Cavitation at the valve could be avoided by opening the valve slowly.

(c) Valve is rapidly closed such that the discharge is halved:

$$\Delta V = -\frac{1}{2} \times \frac{Q}{\pi D^2/4} = -\frac{1}{2 \times} \frac{0.05}{0.7854 \times 0.2^2} = -0.796 \text{ m/s}$$

$$\therefore \Delta p = -1000 \times 1330 \times (-0.796) = \underline{1.06 \times 10^6 \text{ Pa, or } 1060 \text{ kPa}}$$

Final velocity is halved to initial velocity.
Change of velocity is −ve

Compressible flow in an elastic Pipe

Problem 11.49 (Potter)

Oil with a specific gravity of $S = 0.90$ is flowing at 20 ft^3/sec through a 20-in.-diameter 13,000-ft-long pipe. The elastic modulus of the steel pipe is $E = 29 \times 10^6$ lb/in^2, its thickness is 0.40 in. and the bulk modulus of elasticity of the oil is $B = 217{,}000$ lb/in^2. A valve at the downstream end of the pipe is partially closed very rapidly so that a water hammer event is initiated and a pressure wave propagates upstream. If the magnitude of the wave is not to exceed 90 lb/in^2, determine:

(a) The percent decrease of flow rate tolerable during the valve closure.

(b) The time it takes the pressure wave to reach the upstream end of the pipe.

Solution:

Compute the acoustic wave speed:

$$a = \sqrt{\frac{217 \times 10^3 \times 144}{0.9 \times 1.94 \times \left(1 + \dfrac{20 \times 217 \times 10^3}{0.40 \times 29 \times 10^6}\right)}} = 3610 \text{ ft / sec}$$

(a) The allowable changes in velocity and discharge due to pressure constraint are:

$$\Delta V = \frac{-\Delta p}{\rho a} = \frac{-90 \times 144}{0.9 \times 1.94 \times 3610} = -2.06 \text{ ft/s},$$

$$\Delta Q = \frac{\pi}{4} D^2 \Delta V = \frac{\pi}{4} \times \left(\frac{20}{12}\right)^2 \times (-2.06) = \underline{-4.49 \text{ ft}^3 / \text{sec}}$$

Hence the tolerable flow rate decrease is (1 – 4.49/20) × 100 = 77%.

(b) Wave travel time from the downstream to the upstream end of pipe:

$$t = \frac{L}{a} = \frac{13000}{3610} = \underline{3.60 \text{ s}}$$